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Manyfold universe

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Abstract: We propose that our world is a brane folded many times inside the sub-millimeter extra dimensions. The folding produces many connected parallel branes or folds with identical microphysics — a Manyfold. Nearby matter on other folds can be detected gravitationally as dark matter since the light it emits takes a long time to reach us traveling around the fold. Hence dark matter is microphysically identical to ordinary matter; it can dissipate and clump possibly forming dark replicas of ordinary stars which are good MACHO candidates. Its dissipation may lead to far more frequent occurrence of gravitational collapse and consequently to a significant enhancement in gravitational wave signals detectable by LIGO and LISA. Sterile neutrinos find a natural home on the other folds. Since the folded brane is not a BPS state, it gives a new geometric means for supersymmetry breaking in our world. It may also offer novel approach for the resolution of the cosmological horizon problem, although it still requires additional dynamics to solve the flatness problem. Although there are constraints from BBN, structure formation, the enormity of galactic halos and the absence of stars and globular clusters with a discernible dark matter component, we show that the model is consistent with current observational limits. It presents us with a new dark matter particle and a new framework for the evolution of structure in our universe.

Keywords: Field Theories in Higher Dimensions, Extra Large Dimensions, Cosmology of Theories beyond the SM, Physics of the Early Universe.

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Contents

1. Introduction

The idea that we live on a brane embedded in a spacetime with N additional large spatial dimensions may provide an alternative understanding of the hierarchy between the gravitational and electroweak mass scales [1, 2, 3]. In this framework, the fundamental scale of quantum gravity can be as low as $\sim$ TeV, without conflicting with any existing experimental limits. Then the observed Planck mass, $M_P = (G_N)^{-1/2} \sim 10^{19}\,\mathrm{GeV}$, where G_N is the Newton constant, is related to the fundamental Planck scale in $4+N$ dimensions, M_{Pf}, by Gauss law [1]

$$M_P = M_{\mathrm{Pf}} \sqrt{M_{\mathrm{Pf}}^N V_N}\,, \tag{1.1}$$

where $V_N \equiv L_1 L_2 \cdots L_N$ is the volume of the extra space, and L_i is the size of the ith extra dimension. It is clear that this new picture has dramatic consequences both for particle physics and cosmology. In addition to shedding new light on the mass hierarchy problem, in the realm of elementary particle physics the framework of large extra dimensions has tremendous consequences for the low energy phenomenology, accounting for many novel physical phenomena involving fields living in the extra dimensions, such as for example the new origin of approximate symmetries from distant breaking [4, 5, 6], neutrino masses [7, 8], and flavor generation [9] and violation [5] near the TeV scale. It may provide for an alternative understanding of the unification of gauge couplings [10], where infrared effects in the bulk appear as ultraviolet effects on the brane [11, 10]. There are also other new effects such as global charge non-conservation due to production of baby branes [12], and new types of dark matter in the form of super-heavy superstrings [11] or tilted branes [13, 14]. Connections with string theory have been examined in [15]. Explicit solutions of Einstein's equations with large compact spaces were constructed in [16]. In fact, the richness of new phenomena makes a compelling case that theories with large extra dimensions, and a desert in the bulk, are a fully consistent alternative to the conventional theories with a great desert in energy scales, leading to interesting new phenomenological consequences (for an incomplete list of references, see [1, 2, 17, 18] and references therein). They are consistent with present observational constraints, as shown in [1, 2] and confirmed in more detail [19].

Cosmological considerations of models with large extra dimensions confirm that they are a consistent candidate to describe our world. At the first sight it may appear that cosmology poses very difficult challenges to these models, requiring a resolution of cosmological conundrums at energy scales not exceeding $\sim$ TeV. Furthermore, in addition to the usual cosmological problems such as the horizon problem and the flatness problem, there may emerge new problems, involving the possibility of bulk graviton overabundance [2, 20]. However, various aspects of inflation have been considered [11, 20, 21, 22], leading to the proposal of several attractive models, such as the brane inflation of [11, 21] and the asymmetric inflation of [22]. There it has been shown that it is possible for a brane universe to undergo inflation at an early stage, leading to a universe much as our own at low energies. Recent studies have considered some of these and related issues (an incomplete list of references is given in [23, 24]).

In the present paper we will discuss novel astrophysical and cosmological implications of the brane universe with large extra dimensions. We propose that the brane which we live on is *folded* in the bulk of the extra spatial dimensions, producing n_f identical braneworlds, or folds. We will refer to this configuration as the Manyfold. The adjacent folds of a Manyfold are connected by regions where the brane bends and its curvature is very large. We will refer to different folds either as "folds" or "branes" completely interchangeably, while the regions of large curvature where the

Manyfold bends will always be singled out as "tips" or "tips of folds" to avoid confusion. Graphical examples which capture the essential points of these definitions are given in figures 1 and 2.

In the Manyfold worlds, observers on different folds can communicate by exchange of brane-localized degrees of freedom, e.g. electromagnetic waves, but this requires enormously long times, equal to the sum of their distances to the nearest tip of the fold. On the other hand, they can probe objects on adjacent folds gravitationally, since they are nearby in the bulk, at sub-millimeter distances. Hence matter on other folds will appear as dark matter to us. This dark matter would consist of the *very same* Standard Model particles as our own, localized on an electromagnetically distant part of the same brane, and with identical microphysical properties. This property of the folded braneworld appears counterintuitive at the first glance, since gravitationally near objects may appear billions of light years away. However, this is a very natural consequence of the simple features of the model: *the localization* of the Standard Model particles to a *folded* brane which is embedded in large extra dimensions. A low energy observer on the braneworld can therefore experience two different minimal distances: gravitational, defined by minimizing the distances traveled by bulk particles, and electromagnetic, which corresponds to minimizing the distances along the brane, and around the tips of folds.

This picture can shed a new light on a number of cosmological problems, including a new natural dark matter candidate. It suggests important possible new phenomena which we briefly summarize here:

- **Dark matter identical to the normal matter.** This is a consequence of the construction of the Manyfold universe, since the dark branes are continuously connected to ours. Therefore Manyfolds offer a very elegant new approach to the problem of dark matter in the Universe.

- **MACHO stars which do not shine.** MACHO stars made out of dark matter are identical to ours, but shine along other folds, and their light has a long way to go before it reaches us. This can naturally explain why MACHO stars are invisible to us.

- **Hybrid objects.** Dark matter on other folds can clump in the same gravitational wells as the visible matter. This will give rise to hybrid structures with different luminous and gravitating masses and deviations from the expected gravitational effects.

- **Increased occurrence of collapsing systems.** The folded universe predicts more collapsing young structures, which can be subject to observation by gravitational wave experiments.

- **Neutrino mixing.** If the neutrino masses are generated due to the mixing with a higher-dimensional bulk fermion [7], the folded universe picture selects equal mixing between the Standard Model and sterile neutrinos. When there are three folds, the mixing is maximal.

- **Old light.** The *old light* emitted from adjacent folds could reach us after a sufficiently long time, carrying information about the distribution of matter from within, say, our galaxy billions of years ago.

- **Horizon problem.** The folded universe picture permits apparently superluminal communication between different segments of the brane through the bulk. This could give a non-inflationary solution of the horizon problem, if the brane was originally crumpled in a small higher-dimensional box and later unfolded.

  We will also discuss one other important feature of the Manyfold universe which has very important consequences for both cosmology and particle physics, although is not directly related to astrophysical issues:

- **Folded supersymmetry breaking.** A folded brane is not a BPS object, and may therefore be the source of the observed supersymmetry breaking in our world. This gives the simplest realization of the idea of [13], where the non-BPS nature of the brane Universe as source for the observed supersymmetry breaking was first suggested.

The paper is organized as follows. In the next section, we present the basic idea of the folded universe, describing in simple terms the setup within which we will carry out our investigation. In section 3, we discuss the astrophysical consequences of, and limits on, Manyfold worlds. Section 4 is devoted to the discussion of sterile neutrinos which come from the other folds. We will consider aspects of inflationary cosmology in folded universe models in section 5, and explain how inflation can produce Manyfolds. In section 6, we will present the horizon problem from the point of view of superluminal communication through the bulk. In section 7, we will turn our attention to the questions about the stability of the folded brane universe, and folded SUSY breaking. Finally, we conclude with a summary of our results.

2. Bifold

Here we introduce the basic building blocks of the folded universe. For simplicity we start with a folded universe which bends only once in the bulk. From the point of view of a four-dimensional observer this folded universe, or the Bifold, contains two branes which are connected by a region where the master brane bends. A simple pictorial description of a Bifold is given in figure 1. As we have indicated above,

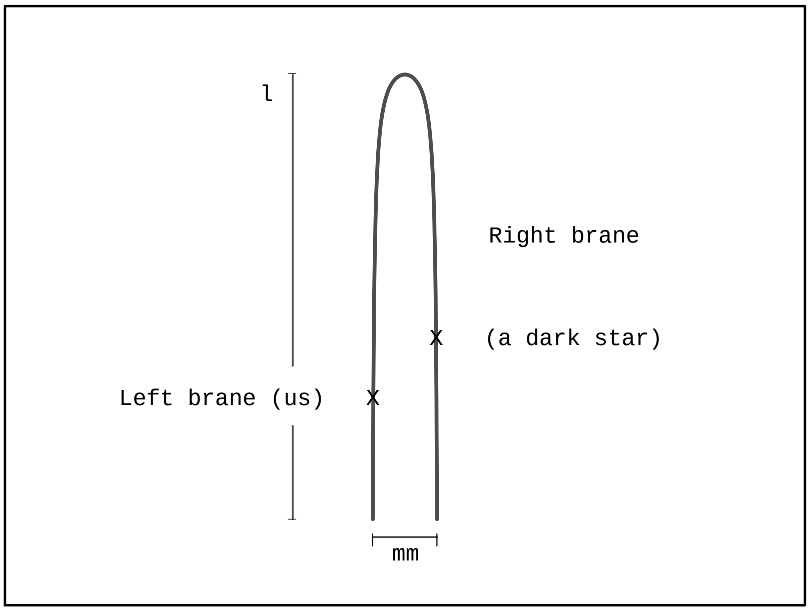

Figure 1: The Bifold universe. The distinction of matter between the Standard Model particles on the left-brane and the dark particles on the right-brane is not microphysical but purely geometrical.

we are using the terminology where we refer to each side of the Bifold as a brane, or a fold, completely interchangeably, whereas to avoid confusion we call the high curvature region the tip of a fold. We will later generalize this to cases with an arbitrary number of folds, or the Manyfold.

Now consider a 3-brane embedded in the bulk with N extra dimensions. Although the lowest energy state belongs to a straight infinite brane, cosmologically such an initial condition is not more likely than any other chaotic distribution of the brane in the bulk. For instance, we can imagine that initial conditions in a spatial region at a high temperature before inflation where given by a brane which was folded n_f times per bulk cross-section. While the orientation of the folds could be completely random initially, this configuration can then be stretched up by a subsequent era of early inflation, and converted into n_f parallel branefold worlds. The difference from other hypothetical brane worlds, which can also be located in the bulk, is that in the present case the matter localized on *all connected* folds will be *identical* by cosmological evolution and the connectedness of the folds.

The local universe seen from any of the individual branefold worlds is rather unusual. The matter localized on the neighboring branes appears as near-by dark matter, since it is less than a millimeter away. However, in contrast to the usual dark matter models, where dark matter particles are completely invisible electromagnetically at low energies, in this case an observer on a fold could eventually see the dark matter if she waits long enough. Indeed, light emitted by the matter on an adjacent fold will propagate along the brane, get refracted by the tip at the end of the fold and finally arrive to the observer's side. She will interpret the long travel time around the tip of a fold by saying that the source was a very distant object. However, this object may in fact be just a millimeter away through the bulk! Thus the dark matter in the halo of our galaxy can be made out of the very same protons and electrons as we are, which could be potentially visible, and we may not even know it!

Of course, the key question is how far the nearest tip of our fold is from us. Obviously, there will be constraints on the longitudinal size of folds, coming from astrophysical considerations, and we will discuss them in detail below. For this purpose, we will imagine that the folded universe consists of n_f folds, which may have their tips at different distances from our shadow on each fold. We will first consider the simplest possibility, which appears to be the most natural in the view of inflationary cosmology: that the tip of the adjacent fold is farther from us than the Hubble radius. Indeed, if the Manyfold has emerged from a chaotic initial distribution of branes by way of, for example, asymmetric inflation [22], then since asymmetric inflation can give may more efoldings along the brane than is necessary to solve the cosmological problems, the tips of folds may be at distances many orders of magnitude larger than the present Hubble horizon. However, we will also generalize this discussion to include the case when tips of folds may be at distances smaller than the present Hubble length. This may come, for example, from branes which were initially extremely wiggly at short distances, and inflation was not long enough to blow the wiggles completely outside of our horizon.

Before we proceed, we address a possible concern one might have about the Bifold (and Manyfold) world. The first thing one might think will go wrong with these ideas is that Hubble's law is violated. For illustration, consider the possibility that we are at a distance from the nearest tip of our fold which is smaller than the present Hubble length. Then we can see objects on the adjacent fold, and in fact can associate two types of positions and velocities to them (see the points depicted on the folds in figure 1.)

(1) in the bulk, denoted $r_{\rm gr}$ and $v_{\rm gr}$, respectively, and

(2) along the brane, $r_{\rm br}$ and $v_{\rm br}$.

This means that there are two Hubble's laws in the Bifold universe, and as long as the relative velocity of the folds is miniscule, it is easy to see that both Hubble's laws are satisfied:

$$v_{\rm gr} = H r_{\rm gr}\,, \qquad v_{\rm br} = H r_{\rm br}\,, \qquad \text{with} \qquad H^2 = \frac{8\pi G_N}{3}\rho_{\rm total}\,, \tag{2.1}$$

where $\rho_{\rm total}$ is the *total* energy density. The relative velocity between the folds can be defined as the difference of velocities each fold relative to the background set

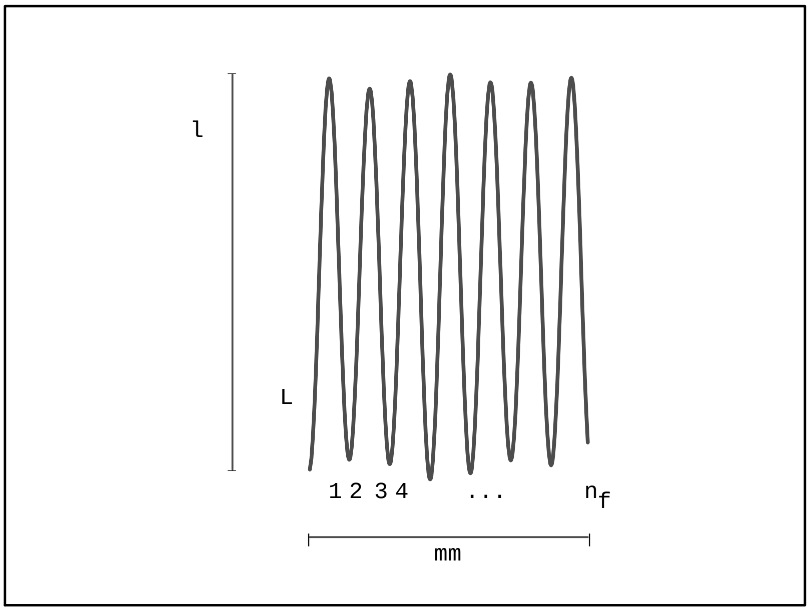

Figure 2: The Manyfold universe. Each side is referred to as a brane, or a fold, while the region where they connect is called a tip of a fold. We live on the leftmost brane L, while there are n_f right-branes in the bulk adjacent to our brane.

by primordial gravitational waves, which give the preferential inertial frame since they are not confined to the folds. These velocities can be measured by determining the anisotropy of the gravitational wave background, and can be considered as a fluctuation in the initial condition of the Bifold, which can be erased by a stage of early inflation. Of the two Hubble's laws, the latter is experimentally checked since we use the Doppler shift of electromagnetic photons which travel along the brane.

3. Astrophysics of Manyfolds

It is straightforward to generalize the Bifold universe to the Manyfold universe, with an arbitrary number of brane folds, as depicted in figure 2. We live on the leftmost brane L and there is a number of "right-branes" labeled by the integer $i = 1, \ldots, n_f$. Here we will discuss astrophysical signatures of a Manyfold. To start with, we will make two simplifying assumptions:

(i) The distance l between us and the closest tip of a fold is much greater than the Hubble length. This was motivated in the previous section.

(ii) The bulk fields, such as the dilaton, have constant *vev*'s. This ensures that the local microphysics that an observer on any brane sees is identical; i.e. the electron to proton mass ratio is the same for all observers, on any of the branes composing the Manyfold.

We will later relax each of these assumptions. Also, for notational simplicity, we assume that the thermodynamic quantities, such as the temperature T_R and the baryon number density n_R, on all the right-branes are identical:

$$T_{i\ R} \equiv T_R\,; \qquad n_{i\ BR} \equiv n_{BR}\,, \qquad \forall\, i = 1, \ldots, n_f\,. \tag{3.1}$$

Now we turn our attention to astrophysical constraints as well as possible observable predictions of the Manyfold universe.

3.1 Big bang nucleosynthesis

The matter and radiation populating the right-branes (R-branes) act as dark matter with properties isomorphic to ours. At early times, its energy density is dominated by the radiation component. The radiation energy density is given by cT_R^4, where T_R is the R-brane temperature and c depends on the number of light species at T_R. The success of Big Bang Nucleosynthesis (BBN) in predicting the abundances of light nuclei demands that there are no more than one extra species of light particles at $T_L = T_{\rm BBN} \lesssim 1\,{\rm MeV}$. This implies that the R-branes are colder than our L-brane, $T_R \ll T_L$. In section 5 we will present an example of how such an asymmetry between the L- and R-branes can result from a small fluctuation during inflation.

3.2 Big galactic halos and the absence of hybrid globular clusters and stars

By the construction of the Manyfold universe, the matter on R-branes (R-matter) has microphysical properties identical to the matter on our own L-brane. In particular, microphysical collisions leading to dissipational processes have identical cross-sections on the L- and R-branes. These dissipational processes are responsible for the formation of condensed structures in the universe, such as galaxies, globular clusters, and eventually stars. In the traditional dark matter models, the dark matter is dissipationless and so it does not form discernible structures smaller than galactic halos. In contrast, R-matter could form smaller structures such as globular clusters and stars, since it dissipates just like our own L-matter. Since the L- and R-matter both cluster around common seeds, which are remnants from an early inflationary era, this raises a disastrous prospect of forming hybrid stars or hybrid globular clusters containing both ordinary and R-matter. Such objects have never been observed. Furthermore, if the R-matter parallels the evolution of normal baryonic matter, it could never be an acceptable dark matter candidate. Since it would dissipate and condense down to galactic size scales, which are far too small compared to the size of galactic halos.

A way around these problems is to postulate that the baryon density n_{BR} on any R-brane is much smaller than our baryon density n_{BL}. This will decrease the rate of dissipation on each of the R-branes, by lowering the rate at which the Rmolecules electromagnetically collide with each other. Indeed, the "cooling time" $t_{\rm cooling}$, that it takes for a plasma to loose a significant fraction of its energy scales as $t_{\rm cooling} \sim n_B^{-1}$ [25]. Since our galaxy formation commenced at a redshift $z \sim 4$, taking

$$\frac{n_{BR}}{n_{BL}} \lesssim \frac{1}{4} \tag{3.2}$$

would ensure that the R-galaxy formation is beginning just now. Therefore, the R-matter has been behaving so far as essentially dissipationless cold dark matter, since it was too dilute to have significantly dissipated its energy via collisions. Indeed, in the limit $n_{BR}/n_{BL} \to 0$, $n_f \to \infty$, with the fixed product $n_f n_{BR} \simeq 30 n_{BL}$, the R-matter would behave *precisely* like the ordinary CDM, critically closing the universe and would form large galactic halos as a result of gravitational clumping. In this limit, the R-matter would not significantly clump on shorter length scales, such as those of globular structures which are known to be the largest objects without a significant dark matter content.

Another reason for the difference in cosmological evolution and structure formation on R-branes is their different light element abundances. Because of these differences, BBN on R-branes will proceed quite differently. Since $T_R \ll T_L$, R-BBN will occur when the energy of the universe and its expansion is driven by the radiation density $\sim T_L^4$ residing on our L-brane. The resulting light nuclear abundances on R-branes will be identical to those of the standard BBN with a large number of massless species $\sim c n_f (T_R/T_L)^4$. These scenarios have been studied by Wagoner [26], and they produce the following outcome. As the effective number of light species $S = c n_f (T_R/T_L)^4$ increases up to $S_{\rm max} \simeq 10$, the ^4He abundance grows until it reaches about $\sim 80\%$ [26]. This occurs because the universe expands so rapidly that neutrons have no time to decay into protons, and they combine with protons to form ^4He. For $S > S_{\rm max} \gtrsim 10$, the ^4He abundance drops because the universe expands so rapidly that ^4He does not have time to form.

Subsequent formation of structure in the universe will depend on the precise chemical abundances of elements. This is clear since dissipation and consequently the cooling time will depend on microphysical cross-sections, molecular masses etc. Although detailed study of structure formation is challenging and beyond the scope of the present work, we emphasize that this is in principle a well-defined program in which, given the ratios

$$(i)\ \frac{T_R}{T_L}, \qquad (ii)\ \frac{n_{BR}}{n_{BL}} \tag{3.3}$$

one can numerically compute first the primordial nuclear abundances and subsequently the details of structure formation inside the R-branes. This should also help us to determine the values of T_R/T_L and n_{BR}/n_{BL} for which we can comfortably form R-stars as candidates for MACHOs, to which we now turn.

3.3 MACHO star formation

An attractive possibility in a Universe of folded branes is the existence of non-shining R-stars, with masses and properties identical to those of our stars, with the exception that they are electromagnetically invisible. Such objects would be natural candidates for the MACHOs which cause gravitational lensing, provided of course that there has occurred sufficient dissipation of R-matter. This requires that the ratio n_{BR}/n_{BL} is not much smaller than 1/4, so that the cooling time t_{cooling} is of the order of the Hubble time. Then, the R-matter dissipation would be significant enough to allow the formation of R-stars.

There is a tension between R-star formation, which places a lower limit on the ratio n_{BR}/n_{BL}, and the absence of hybrid globular clusters or the existence of big galactic halos, which favor small values of n_{BR}/n_{BL}. It is beyond our scope to be more precise on the lower bound on n_{BR}/n_{BL} implied by the requirement of R-MACHO formation, than to say that $n_{BR}/n_{BL} \sim 1/4$ would imply that R-dissipation is commencing now. Perhaps such a value of n_{BR}/n_{BL} is large enough for some small but potentially interesting and measurable contamination of globular clusters or even primordial POP III stars by clumped R-matter. Note that gravitational lensing data favor non-shining MACHOs of sub-solar mass, which as R-stars are a natural consequence of the Manyfolds.

3.4 Enhanced gravity waves at LIGO and LISA

An important difference between CDM and R-dark matter (R-DM) is that the latter can rapidly loose energy and violently collapse, creating gravitational waves. This increases the number of gravitational wave sources relative to the CDM models. If the number of violent astrophysical processes happening today on the R-branes is of the same order as those happening on our brane, then the total number of gravity wave sources increases by a factor ~ 30 — the ratio of DM to ordinary matter. In the case where our distance l from the nearest tips of folds is less than the Hubble length, the signal of R-sources would be further enhanced by their close proximity to us by an additional factor of order of $\sim (lH)^{-1}$. This leads us to the discussion of constraints and signatures in Manyfolds where $l \ll H^{-1}$. This corresponds to relaxing the assumption (i) stated at the beginning of this section.

3.5 Astrophysics near the edge of the fold

The case when $l < H^{-1}$ offers an exciting possibility that an object which appears to be very far — electromagnetically — is gravitationally very close, perhaps even comprising the dark matter of our very own galaxy! We have touched upon this possibility in section 2, and here we will consider it more closely.

An immediate consequence of the identification of distant objects with the nearby dark matter is that violent astrophysical processes which appear to be happening very far away may in fact have occurred in our galactic neighborhood and have given rise to much stronger gravitational waves than inferred from the electromagnetic distance determined by spectral Doppler shifts. As mentioned, this would result in an enhancement of gravity wave signals at LIGO and LISA of order $\sim 30(lH)^{-1}$.

Another potential signature would be an apparent violation of Newton's force law at large distances, since the gravitational force would no longer scale as the inverse square of the *electromagnetic distance* of two particles on different folds. Of course such violations of Newton's force law are not necessarily undesirable since the very postulate of the existence of dark matter is forced upon us because of an inconsistency between Newton's law and what we literally "see".

However, before embarking on a detailed investigation of these *exotica*, we ought to address possible difficulties with the case $l < H^{-1}$. This has to do with the requirement that electromagnetically distant parts of our universe are located in others folds, where $T_L \gg T_R$, and hence are cooler now. Even more importantly, they have been much colder during BBN. This implies that they must have different primordial light element abundances. In particular, the primordial deuterium abundance at large distances $d > l$ should be different than the local abundance. But, the deuterium abundance has been measured out to cosmological distances of a fraction of H^{-1} inside Hydrogen clouds by looking at quasar absorption lines. This at face value suggests that l is at least a fraction of unity of H^{-1} or larger. On the other hand, distant D-abundance is just one number (until recently controversial at that) and one could imagine reproducing it in other ways, such as changing n_{BR} appropriately. This of course could create different observational problems, namely distant net baryon abundances n_{BR} which are different from the local one n_{BL}. Again, such variations in $n_{BR} \neq n_{BL}$ may be acceptable at cosmological distances. It seems that the safest choice is to take l to be larger than a factor of H^{-1}, although we have not actually found a concrete limit but possible problems with smaller l's.

3.6 Profiles in the bulk

If bulk fields do not vary through the bulk, microphysics on each fold is identical. However, bulk fields generically can change in the bulk (see e.g. [27] for a review), developing a non-trivial profile. For example, branes provide sources for the moduli, which therefore develop non-trivial profiles in the bulk. Due to Gauss Law, typically these profiles behave as $a + b\exp(-rm)/r^{N-2}$ at large distances, where m is the mass of the bulk field, satisfying a very weak constraint $M_{\rm Pf} > m > 10^{-2}\,\mathrm{eV}$, where the lower bound comes from the fifth force searches. If either m or N is large, the profile will rapidly approach a constant value far from the source brane. Otherwise for a small mass and e.g. $N = 2$, the field may even approach a logarithmic shape. An immediate consequence of having a non-trivial bulk field profile is that since the dilaton *vev* sets the values of gauge couplings on the brane, different copies of the Standard Model on each folds will have different strengths of couplings depending on where they are in the bulk. This produces a natural asymmetry between branes, and can give rise to different cosmological evolution of different folds even if the initial temperatures after inflation on different folds were equal.

Indeed, if for example the electron weighs as much as the proton on the R-branes, then all R-atoms will have very small dipole and multipole moments $\sim (e/m_p)^\alpha$, and so R-matter will be essentially dissipationless, just like CDM. Another possibility is that the u quark may be heavier than the d quark, and the lightest stable baryon is the neutron, impeding the formation of R-atoms. Still another possibility is that just the value of the canonical dilaton field is varying in the bulk, leading to an overall scaling of couplings.

Since the formation of structure and stellar evolution are so sensitive to the masses and couplings of elementary particles, even a small variation can lead to drastic changes and therefore makes them difficult to track down in detail. Because there is no well-motivated and concrete model with a solution having specific bulk-varying fields, we will continue to study the simplest possibility of constant bulk fields. In fact, the hypothesis of constant bulk fields by itself and on very general grounds necessarily implies the existence of multiple identical copies of our brane, without the need for a physical connection between the branes. This is because the constancy of the bulk fields requires the simultaneous presence of D-branes (positive sources) and coincident Orientifolds (negative sources). The latter can only occur in multiple copies in compact spaces. In simple cases, such as toroidal compactifications, they come in 2^N identical copies, where N is the number of new compact dimensions. In this case, each copy would support locally identical particle physics although there is no direct physical contact between different branes, but the phenomenology would still be the same as in the example of a folded brane, subject to identical astrophysical and cosmological constraints.

To conclude this section, we point out that our discussion shows that mirror models with mirror particles isomorphic to ours, as for example those of [28], are excluded on simple grounds if they compose more than 1/30 of the critical density. They dissipate more than ordinary matter and would form halos smaller than our galaxy. They would also strongly mix with matter on shorter length scales leading to hybrid globular clusters of hybrid primordial stars. To avoid such disasters, one would have to postulate ~ 100 identical mirror models. Alternatively, postulating an asymmetry between our world and the mirror world is possible, and can be generated naturally in Manyfolds with profiles in the bulk. In that case, the mirror particles can have different microphysical properties, and evade the astrophysical constraints with employing fewer copies.

4. Sterile neutrinos from distant folds

The Manyfold universe allows for a very peculiar neutrino mixing for the models of neutrino mass generation *a ĺa* [29] employing the higher-dimensional mechanism suggested in [7]. Rather then reviewing the details, here we will consider how this picture changes in the case of the Manyfold universe. Consider first the other side of the fold across our brane. The "image" of our left-handed neutrino becomes a right-handed particle. This is due to the fact that the brane changes its orientation. As a result, all the localized zero modes flip their chirality, where the left-movers become right-movers and visa versa. This reflects the fact that a brane becomes an anti-brane upon folding (see the discussion in section 7).

To see this in more detail, we consider an explicit example of a kink (anti-kink) brane (which we will discuss in more detail in section 7). A kink (anti-kink) brane is supported by a scalar field which interpolates between two degenerate and disconnected vacua, $\phi = \pm v \tanh(\lambda v x_5)$, where λ and v are its coupling constant and *vev*, respectively. Let now $\mathcal{N}$ be a five-dimensional Dirac fermion coupled to this scalar and let us consider Dirac equation for this fermion in the kink (or anti-kink) background:

$$(i\Gamma^\mu \partial_\mu + i\Gamma_5 \partial_5 - \phi)\,\mathcal{N} = 0\,. \tag{4.1}$$

It is well known that this equation admits only one localized chiral mode

$$\mathcal{N}_{L,R}(x_5, x_\mu) = \mathcal{N}_{L,R} \mathrm{e}^{\pm \int_0^y \phi_(y) dy}\,, \tag{4.2}$$

where the chirality is defined by the boundary conditions. The reason is that in infinite extra dimensions there is only one normalizable mode for a specific choice of the background, as is clear from the exponential factor in (4.2). Alternatively, when the bulk is compact, only one of the modes is dynamical while the other effectively decouples. Therefore, if the brane localizes the left-handed fermion, the other side of the fold can only localize the right-handed one.

In the case when a brane and an anti-brane are connected, the right-handed neutrino which is localized on the other side can be thought of as our left-handed neutrino which changes chirality after refraction around the tip of our fold. The impact of this observation on the structure of the neutrino mass matrix is that now the zero mode of the left-handed bulk state $\mathcal{N}_{0L}$ is not decoupled anymore, as in [7], but mixes with the right-handed image ν_R on the anti-brane, and becomes massive. The mass matrix can be written as:

$$\bar{\nu}_L M \mathcal{N}_R\,, \tag{4.3}$$

where $\nu_L^T \equiv (\nu_L, \tilde{\nu}_{1L}, \tilde{\nu}_{2L}, \ldots)$ and $\mathcal{N}_R^T = (\mathcal{N}_{0R}, \tilde{\mathcal{N}}_{1R}, \tilde{\mathcal{N}}_{2R}, \ldots)$; the modes $\mathcal{N}_{0L}$, $\hat{\mathcal{N}}_{nL}$ $\hat{\mathcal{N}}_{nR}$ decouple from the system. The mass matrix M for any $k+1$ low-lying states is

$$M=\begin{pmatrix} 0 & m_D & \sqrt{2}m_D & \sqrt{2}m_D & \cdots & \sqrt{2}m_D \\ m_D & 0 & 0 & 0 & \cdots & 0 \\ \sqrt{2}m_D & 0 & \frac{1}{R} & 0 & \cdots & 0 \\ \sqrt{2}m_D & 0 & 0 & \frac{2}{R} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \sqrt{2}m_D & 0 & 0 & 0 & \cdots & \frac{k}{R} \end{pmatrix}. \tag{4.4}$$

Now, it is straightforward to generalize this structure for an arbitrary number of folds. We refer to the L-brane, that we reside on, as the $0th$-brane, and count the R-branes in the usual fashion (see figure 2). With this enumeration, the electroweak doublet neutrinos which are localized on even-numbered branes $n_{\rm brane}=0,2,4,\ldots,2k$ will be left-handed for observers on the L-brane, $(\nu_L^{(0)},\nu_L^{(2)},\ldots,\nu_L^{(2k)})$, just like in the Standard Model, while the neutrino doublets localized on odd-numbered branes $n_{\rm brane}=1,3,\ldots,2k+1$ will be right-handed, $(\nu_R^{(1)},\nu_R^{(3)},\ldots,\nu_R^{(2k+1)})$. Therefore we see that the zero mode of the right-handed bulk neutrino $\mathcal{N}_{0R}$ will pair up with all the left-handed brane modes with *exactly equal* mass terms:

$$m_D\bar{\mathcal{N}}_{0R}\left(\nu_L^{(0)}+\nu_L^{(2)}+\cdots+\nu_L^{(2k)}\right), \tag{4.5}$$

and similarly, the left-handed zero bulk mode $\mathcal{N}_{0L}$ will pair up with the right-handed brane states. This implies that an observer on our L-brane will perceive all the states $n_{\rm brane}\neq 0$ as sterile neutrinos. Thus this scenario predicts the equal mixing between our ν_L and all sterile neutrinos, with the mixing angle determined only by the number of folds. How many modes become massive depends on the number of higher-dimensional species in the bulk. In the simplest case of a single bulk neutrino $\mathcal{N}$, the only massive combination is mostly

$$\left(\nu_L^{(0)}+\nu_L^{(2)}+\cdots+\nu_L^{(2k)}\right), \tag{4.6}$$

while the others remain massless. Note that in the case of Threefolds, the mixing mechanism is maximal. Otherwise, maximal mixing may be generated by profiles in the bulk, where the mixing angle would depend on the bulk field in addition to the number of folds.

Since this picture predicts a number of new, light, sterile neutrinos with equal mixing, the nucleosynthesis bounds discussed in [8] should obviously be reconsidered. The reason is that now $\nu_L^{(2k)}$ can be produced by neutrino oscillations. Hence the revised nucleosynthesis bound reads [30]

$$\Delta m^2<\frac{3\cdot 10^{-5}\,\mathrm{eV}^2}{\sin^4 2\theta}. \tag{4.7}$$

Since in our case $\sin^4(2\theta)\sim 1$, this is essentially a bound on m_D, giving $m_D\lesssim 10^{-2}\,\mathrm{eV}$.

5. Initial conditions for the Manyfold

We have seen in the preceding discussion that one of the main constraints on the Manyfold universe is that the baryon number density on our L-brane is substantially larger than on all the R-branes. At first sight, this may appear as a serious fine tuning problem. However, we will argue that, in fact, inflationary cosmology may naturally produce such an asymmetry provided the inflaton is a wall-localized field.

In order to simultaneously produce the four-dimensional Planck mass M_P according to (1.1) and to generate the density perturbations in the range required by COBE, inflation in the scenarios with a very low Planck scale should occur before the extra dimensions are stabilized. A very natural candidate for the bulk inflaton is the radius modulus (radion) [22]. A difficulty with any generic scenario of bulk-field-driven inflation is that it usually produces a Universe with a cosmological moduli problem, because after the exit from the early stage of inflation, the universe is dominated by coherent oscillations of a weakly-coupled modulus. Another possible objection is that bulk inflaton could reheat the whole bulk, which could lead to the overproduction of bulk gravitons [1]. In the asymmetric inflation scenario however the latter problem is easily avoided, as was discussed in [22]. However, these problems can be solved by a secondary stage of inflation, which appears to be necessary regardless of the early cosmological history [11, 21].

A viable scenario can then unravel as follows. First, the asymmetric inflation can start in a region of the bulk where the brane is very curved. Then, rapid expansion in the longitudinal directions will stretch and parallelise the folds of the brane, producing, generically, a Manyfold whose tips are at distances much larger than the cosmological horizon at late times. If the period of asymmetric inflation is short, then the tips may be at distances comparable to the scale of the Hubble horizon. This stage of early asymmetric inflation however would produce the same matter energy density on all folds, and a Manyfold dominated by its radion modulus. A subsequent stage of wall inflaton-driven inflation can then produce the situation where a single fold ends up with the highest reheating temperature, and the largest baryon number density, whereas the remaining folds turn out to be much colder. Indeed, let Φ be a scalar field localized on the brane, with the potential of this field $V(\Phi)$ which has a region flat enough to satisfy the slow-roll conditions. If Φ is initially displaced from the minimum, it can drive inflation after the stabilization of the extra dimensions. Since it couples to the Standard Model particles on the brane with non gravitational, renormalizable couplings of the form

$$g\Phi\Psi\Psi\,. \tag{5.1}$$

after inflation ends, Φ will begin to oscillate around the minimum and decay into the fields Ψ. If the coupling constant g is large enough, the decay will be rapid enough producing a universe with normal relativistic particles, no moduli problem, and the reheating temperature high enough for nucleosynthesis to proceed.

When there is $n_f + 1$ similar folds, then each fold supports an inflaton which couples to a replica of the Standard Model on its fold according to eq. (5.1) and only *gravitationally* to the copies of the Standard Model residing on the other folds. Since gravity can propagate through the bulk, each individual inflaton can simultaneously drive inflation on all of the branes, but can reheat efficiently only *its own* brane. Since the wall inflatons start out with different initial values, which can in fact be set up naturally by a stage of asymmetric inflation of [22], which stretches the spatial fluctuations of wall fields to very large distances. Hence the fluctuations whose wavelength is comparable to the size of the fold will be seen as wall fields lying at different distances from the minimum of the potential. Therefore they will reheat their respective folds at different times. In general, all the reheating energy on the folds where the inflaton stepped out of the slow roll early will be redshifted away by inflation driven by inflatons on the other branes, until the last inflaton relaxes to its minimum. The observable energy density in the Universe will come predominantly from this last reheating, and we denote the inflaton responsible for this as $\Phi_{\rm our}$. As we discussed above, it couples directly to fermions on its own brane

$$g\Phi_{\rm our}\psi_{\rm our}\psi_{\rm our} \tag{5.2}$$

and gravitationally to the fermions on the other branes

$$g'\Phi_{\rm our}\psi_{\rm other}\psi_{\rm other}\,. \tag{5.3}$$

Here g' is a dimensionless ratio of the scalar *vev* of $\Phi_{\rm our}$ to the Planck mass, mimicking gravitationally suppressed couplings, $g' \ll g$. Although the energy density released on "our" brane $\rho_{\rm our}$ is controlled by g, while the fraction released on any other brane will be suppressed by inverse powers of $M_{\rm Pf}$, the total energy density released on other branes may still exceed the energy density $\rho_{\rm our}$ on our brane, since the multiplicity of channels can over-compensate gravitational suppression, depending on the ratio g/g' and the number of folds. Assuming that this in fact happens, although the baryons on any other brane are fewer in number, their integrated energy density dominates over the energy density on our fold. This explains our choice of terminology, where we refer to the fold with the longest-lasting inflation as "ours", since it is the only brane that could support life so far!

A very simple model of wall inflation is the brane inflation [21], where the inflaton is the inter-brane separation whose variation is generated by the motion of branes in the bulk. Recalling that the branes are the D-branes of string theory, the brane on which we reside is really a stack of m parallel D-branes, generating an unbroken SU(m) gauge symmetry on the brane. This should not be confused with the $n_f + 1$

folds, which are all parts of this same stack, supporting the geometric "mirrors" with precisely the same $\mathrm{SU}(m)_n$ symmetry. In the early Universe, the slices can be randomly displaced and start falling towards each other. If Φ is identified with the field which parameterizes the inter-brane distance r, $\Phi = M_{\mathrm{Pf}}^2 r$, then the motion of branes is equivalent to the rolling of the scalar field Φ_m in the adjoint representation of $\mathrm{SU}(m)$, whose vacuum energy drives inflation [21]. The inter-brane potential is sufficiently flat to drive inflation, since it is generated by the bulk closed string exchange, which gives rise to terms with inverse power-law behavior at distances $r \gg M_{\mathrm{Pf}}^{-1}$. When SUSY is broken, a good approximation for the potential is [21]

$$V(r) = M^4 \frac{a(M_{\mathrm{Pf}} r)^{N-2} + b_i e^{-m_i r} - 1}{(M_{\mathrm{Pf}} r)^{N-2}}\,, \tag{5.4}$$

where M is the brane tension, a, b_i are some constants, and m_i are masses of the heavy bulk modes (e.g. RR or dilaton fields, if these are massive).

Hence a natural scenario for brane inflation in the Manyfold universe is to take a randomly distributed gas of branes as the initial condition, produced by asymmetric inflation [22]. These branes can fall towards their equilibrium points, and during their motion longitudinal inflate exponentially. The branes reheat when they collide, but the reheating products redshift away as long as any pair of branes remain in motion, straggling behind the rest. This very last moving branes will drive the last stage of inflation and be responsible for the final stage of reheating, ending up as the branes with the highest baryon content. These are the branes where we can exist. After reheating, subsequent cosmological and astrophysical history will unravel according to the discussion in the previous sections.

We should stress here that the microphysical coincidence of the dark matter particles with the Standard Model ones is a direct consequence of the Manyfold world which emerges from inflation. This is one of the key differences of our framework from any conventional models with mirror dark matter [28]. Here we are *not* postulating the existence of any otherwise arbitrary "mirror" worlds which happen to accommodate the same microphysics as our world. As we have discussed above, the mirror worlds are *created* by the cosmological dynamics, evolving from the initial conditions which are set up after inflation and are parts of our own brane.

6. The horizon problem

In a Manyfold universe, different folds can communicate at rates which appear superluminal to a brane-localized clock, composed of the Standard Model particles. Such signals must propagate through the bulk, making short-cuts through the extra space. Hence different folds could correlate faster through the bulk than by the emission of light pulses traveling along the brane. However these effects do not violate causality of the theory. An observer on the fold, ignorant of bulk gravity, might mistake the information carried by bulk gravitons as a causality violation, but this is merely an illusion, since once all degrees of freedom are accounted for, the theory is local and causally well behaved.

This offers a non-inflationary possibility for solving the cosmological horizon problem. Indeed, consider a 3-brane large enough to contain the comoving volume of our observed universe, but initially very densely crumpled to fit inside a small region of the bulk. The reason the brane can fit inside such a small volume is that it is a smooth manifold of lower dimension, and hence it occupies only a small portion of the bulk (the lower bound on the bulk volume into which a brane can fit is set by the thickness of the brane d and its linear dimension L, and is $V_{\rm bulk} \gtrsim L^3 d^N$). The brane can dynamically unfold later. By "unfolding" we mean that while most of the crumpled parts will straighten out, there will still remain a number of large parallel folds with different temperatures and the size equal to or larger than the comoving horizon scale. If the duration of unfolding τ is sufficiently long so that different parts of a fold come into causal contact, the horizon problem would be solved, since the background radiation can thermalize. Thermalization can proceed via two different scenarios, the "gravity-mediated" scenario, where the information is conveyed by bulk gravity, and the "gauge-mediated" scenario, where the information carrier is a brane-localized gauge boson, e.g. electromagnetic waves. The main difference between the latter case and the conventional cosmology is that due to brane dynamics, the evolution of the causal structure of the crumpled brane is halted by brane dynamics for the duration τ of this stage, permitting the regions of a fold to thermalize.

The process for unfolding the brane is clearly the main ingredient of the solution. A simple dynamical model which provides insight into brane dynamics may be constructed as follows. At large distances the dynamics of a 3-brane is governed by the Nambu-Goto action

$$S_{\rm brane} = \int d^4x \sqrt{-g} M^4 \,, \tag{6.1}$$

where for simplicity we have taken a 3-brane moving in some fixed bulk background. The coordinates x parameterize the world-volume of the brane, $g_{\mu\nu}$ is the induced metric on the brane, and M^4 is the brane tension, which we treat as a free parameter. We will ignore gravitational back reaction since the main effect of gravity is to cause the brane to collapse, and this can be consistently modeled by the action (6.1) alone. Namely, dynamics of the brane is defined by extremizing this action. If we consider an almost flat, infinite brane, then the solutions to the extremal problem correspond to the brane moving with an arbitrary constant velocity, in perfect analogy with the motion of a free relativistic particle. However, if we consider a spherical brane, the action (6.1) contains an instability which leads to brane shrinking. This is because of the curvature of the brane: for example, consider a spherical 3-brane of radius R moving in a flat $5D$ spacetime. In this case, the action (6.1) reduces to $S = -2\pi^2 M^4 \int dt R^3 \sqrt{1-\dot{R}^2}$, and the ensuing Euler-Lagrange equations reduce to a single equation

$$\dot{R} = \pm\sqrt{1 - CR^6}\,, \tag{6.2}$$

where C is a constant of integration. This shows that starting with any initial velocity, the brane can expand up to the maximum radius $R_{\max} = C^{-1/6}$, after which it collapses back to zero size. The instability will persist for any sufficiently crumpled brane. On the other hand, in order to solve the horizon problem the brane must be crumpled into a very small bulk volume, where the instability would try to prevent the brane from ever unfolding.

The brane can resist the collapsing instability if there is conserved non-vanishing particle number (i.e. chemical potential) on the brane. Indeed, let there be a thermal bath of particles localized on the brane, e.g. the Standard Model fermions, at a high temperature. At first we can consistently neglect the cooling by bulk graviton emission, and assume that the entropy on the brane is conserved. Consider now a segment of a crumpled brane, and again for simplicity let the segment be spherically symmetric with the radius R. Ignoring the motion of the brane through the bulk to the lowest order, roughly the quantity which governs dynamics is

$$F \sim (M^4 + T^4)R^3\,, \tag{6.3}$$

where T is the temperature. In the absence of particles on the brane, the segment would collapse, in agreement with the results above. However, if $n \sim T^3 \neq 0$, the gas has pressure on the brane, which opposes its collapse. Indeed, minimizing (6.3), subject to the condition of constant entropy

$$TR = \text{constant}\,, \tag{6.4}$$

we get the minimum size for the curvature

$$R_{\min} \sim \frac{1}{M} \tag{6.5}$$

and the corresponding temperature

$$T_{\text{average}} \sim M\,. \tag{6.6}$$

Therefore we see that in the absence of gravity a crumpled brane filled with a heath bath at some high temperature T_{initial} will reach a state of minimal radius and *uniform* temperature. This means, the brane will thermalize spontaneously by evolving to this state! At this point the brane becomes effectively *tensionless*, and it does not collapse any further. We note that this simple toy model is very similar to a star being stabilized by radiation pressure against gravitational collapse. Indeed, there too the temperature of a star becomes essentially homogeneous with the exception of perturbations. To push the similarity a little further, we could compare the process of unfolding the crumpled brane with a star going supernova.

In a more realistic situation, the brane will cool by dynamical effects. There are two main model-independent channels for cooling: (1) evaporation the bulk gravitons [1] or baby branes [12] into the bulk, if it is colder than the brane; and (2) cosmological expansion, when gravity is turned on. We will ignore the former effect and focus only on the latter, which corresponds to adiabatic cooling.[1] The idea here is that the expansion of the universe will only begin to influence the evolution after a time τ, needed for the Universe to expand by a factor L/D. If this coincides with the time needed for a tensionless crumpled brane of size V to unfold, during which the temperature of the brane changes very little, and the brane remains thermalized so that immediately after this epoch the Universe can expand according to the usual four-dimensional FRW model. Hence $T \sim M$ is the *normalcy* temperature of the universe, defined in [1] as the temperature below which the cooling by bulk graviton emission is negligible. Numerically, to make sure that BBN can proceed normally, we must require that $T \sim T_* > \mathrm{MeV}$.

To gain a more quantitative description of these phenomena, we consider a 3-brane of volume $V_3 = L^3$ which is crumpled into a small fraction of the bulk, of volume $V_{3+N} = D^{3+N}$. If the box size is much smaller than the longitudinal size of the brane, $D \ll L$, we can solve the horizon problem, provided that L is equal to or larger than the comoving horizon size today, and D is of the order of the Hubble length at the time scale set by the brane temperature. Then different parts of the 3-brane can thermalize if either of the following two conditions holds:

$$\begin{aligned} &L \sim L_*\,, \qquad \text{and} \qquad D \sim H^{-1} \qquad \text{or} \\ &\tau \sim L_*\,, \end{aligned} \tag{6.7}$$

where L_* is the comoving scale of the present Hubble size evaluated at the temperature T_* in a *conventional* FRW Universe, and H is the *effective* Hubble parameter at that time. These two conditions correspond to gravity-mediated and gauge-mediated scenarios, respectively.

Unfortunately the conditions (6.7) are very hard to satisfy. Indeed, the minimal amount of energy of a crumpled brane is $\sim L_*^3 T_*^4$. This must be squeezed into a higher-dimensional box of volume $V_{3+N} \sim D^{3+N}$. Therefore the higher-dimensional Hubble parameter which corresponds to the energy density $\rho \sim L_*^3 T_*^4/D^{3+N}$ is $H^2 = L_*^3 T_*^4/(D^{3+N} M_{\mathrm{Pf}}^{2+N})$. But in fact let us assume for simplicity that the size of extra dimensions is stabilized (it can be easily shown that relaxing this condition does not alter the conclusions). This is self-consistent only if $H^{-1} > R$, otherwise the bulk

[1] Evaporation of baby branes requires some clarification in the present context. Normally this process is exponentially suppressed, $\sim \mathrm{e}^{-E/T}$, where E is of the order of the energy of a baby brane [12]. In the present case, since the brane is effectively "tensionless" once the equilibrium is reached, $M \sim T$ and the production of tiny baby branes may be unsuppressed. However, note that even if it is unsuppressed, this process will not significantly alter the entropy of the brane, since a baby brane of the size R^3_{baby} can only take entropy $\sim T^4 R^3_{\mathrm{baby}}$.

would fragment into many disconnected regions of Hubble size. Since the transverse size of a region is then bounded by $\sim R^N$, the total volume is transmuted into $D^{3+N} \to D^3 R^N$, and we get

$$H^2 = \frac{L_*^3 T_*^4}{D^3 M_P^2} = \frac{L_*^3}{D^3} H_*^2 \,, \tag{6.8}$$

where $H_*^2 = T_*^4/M_P^2$ is the corresponding normal four-dimensional Hubble parameter in an FRW Universe at the temperature T_*, and we have used eq. (1.1) to eliminate $M_{\rm Pf}$. Since D must be at least as large as H^{-1}, the Hubble parameters must satisfy

$$H^{-1} = L_* (L_* H_*)^2 \,. \tag{6.9}$$

In a $4D$ FRW universe $L_* H_* > 1$, and hence the above expression can hold only if $T_* \sim 3^o$K, meaning that we cannot fit an arbitrarily large brane into a given higher-dimensional Hubble volume: pushing the brane in makes the volume smaller! This is because the brane must carry entropy necessary to protect it against the instability which causes collapse. A loophole in the above argument could be provided by a brane which is effectively tensionless, and with only a small amount of entropy deposited on it. If this is the case, then it may be possible to thermalize the crumpled brane at a higher temperature, however such solutions are not known at present. The second condition (6.7) requires a slow unfolding of the brane. In the brane unfolds due to the expansion of the universe, the time it takes is

$$\tau \sim \frac{1}{H_*} \,. \tag{6.10}$$

Again, if this is to satisfy the condition (6.7), the temperature must be $T_* \sim 3^o$K, which again is too low.

Both of these mechanisms point toward the same value of T_*, but unfortunately at temperatures far below MeV. Hence while it is conceivable that the horizon problem may be solved, at present this mechanism does not give a universe which is homogeneous at temperatures hot enough for BBN to proceed unhampered. Furthermore, this process also does not address the flatness problem, which would become acute when gravity is turned on and general initial conditions in the bulk and on the brane are allowed. Namely, the temporary resistance of the brane to collapse generated by entropy deposited on it might suggest that even flatness can be addressed. However, when gravity is turned on, it will lead to stronger attraction, since the energy carried by the gas on the brane will add to the gravitational binding, and possibly disrupt the balance which kept the brane from imploding. Nevertheless, it is interesting to note that at least the horizon and the flatness problems might be somehow decoupled from each other. Further, the difficulty in solving the horizon problem with the crumpled brane scenario above arises in the few simple cases which we have considered. If there are certain binding interaction, which would slow down unfolding, the problem could be avoided. It would be interesting to see if such a mechanism could be provided by long strings which can stretch between different parts of the brane.

7. Stability of the folded brane and folded SUSY breaking

When a brane folds, its two sides behave as a brane and an anti-brane (ignoring the tip of the fold). For instance if the brane in question is a D-brane, the two sides will carry opposite RR charge.[2] Hence in general the two sides can attract, and could in fact annihilate. This questions the stability of the Manyfold universe, and requires some clarification. The brane-anti-brane correspondence between any two adjacent folds can be simply understood by noting that the world-volume element to which an RR form couples,

$$S_4 = \mu \int d\sigma C_{(4)}\,, \tag{7.1}$$

changes sign after folding. This happens because the folding corresponds to changing the orientation of the brane, which is induced by the reflection of one of the coordinates on its world-volume. Another way to understand this is to note that the charge of a closed brane must be zero. Indeed, the charge of a Dp-brane embedded in a spacetime with N transverse dimensions can be estimated by evaluating the integral from the dual of a RR field strength around a closed $(N-1)$-dimensional surface which encircles both folds in the transverse space

$$\int_{S_{N-1}} *F_{(p+2)}\,. \tag{7.2}$$

For a closed brane, this surface can be deformed and shrunk to a point, which shows that the integral eq. (7.2) must vanish and so the RR charge must vanish too.

Therefore this brane-anti-brane pair is not a BPS state any more, because now the force mediated by the exchange of the RR gauge bosons adds to the dilaton and graviton attraction instead of compensating them. The system becomes unstable, with a tendency to collapse. On the other hand, the non-BPS nature of the folded Universe may even be welcome, since it would break supersymmetry in the real world. This is in the spirit of the approach of ref. [13], where it has been suggested that non-BPS walls may be the source of the observed supersymmetry breaking. However, to realize this possibility in the present context, one must develop ways to stabilize the Manyfold structure, cancelling the overall attraction between the folds. In this section, we consider examples where a brane-anti-brane pair can be stabilized at some finite separation due to additional bulk states in the theory.

[2]We thank Nathan Seiberg and Massimo Porrati for useful comments on this issue.

Consider a simple field theoretical example of a brane embedded in a space with a single transverse dimension. Such a brane can be modeled by a domain wall generated by a real scalar field with a discrete symmetry breaking potential:

$$V = \frac{\lambda^2}{2}(\phi^2 - v^2)^2 \,. \tag{7.3}$$

This system has topologically stable solutions, exemplified by the kink/anti-kink configurations:

$$\phi = \pm v \mathrm{th}(\lambda v x_5)\,, \tag{7.4}$$

where $\pm$ refers to the kink and the anti-kink, respectively. Viewed separately, each of these solutions is stable due to non-vanishing topological charge. However, a kink-anti-kink pair has vanishing total charge and therefore is unstable. The configurations with vanishing total charge can be stabilized if there are other bulk fields. For instance let us introduce another real field θ defined modulo 2π: θ can be thought of as the phase of a complex field with a non-vanishing *vev*. This means that the scalar field potential can now be written as

$$V = \cos\theta(-a - b\phi^2) + \frac{\lambda^2}{2}(\phi^2 - v^2)^2\,, \tag{7.5}$$

where we could have added many other terms compatible with the symmetries $\phi \to -\phi$, $\theta \to \theta + 2\pi$, without altering any of our conclusions below. However, for the sake of simplicity we will restrict our attention only to the potential of the form (7.5), since it is sufficient for the purposes of the argument. Now in the presence of the field θ there arise two types of topological defects, as can be seen by simple topological considerations: kink (anti-kink) given in (7.4) (where v should be renormalized to account for the additional terms in the potential) and sine-Gordon solitons, the simplest of which are those along which the field θ changes by 2π when x_5 goes from $-\infty$ to $+\infty$. Let us compactify the fifth dimension on a circle and consider the case with two solitons and a kink-anti-kink pair. Note that since the periodicity of θ is consistent with periodic boundary conditions, there may be an arbitrary number of solitons. However, the total topological charge of the branes on the circle must vanish, meaning that there should be an equal number of branes and anti-branes.

We now seek the equilibrium state for such a system. In the absence of gravity, the dynamics is governed by the following three types of interaction: (1) a short range attraction between the kink and the anti-kink, with the strength set by λ; (2) an analogous repulsion between solitons set by a; and finally, (3) an attraction between a soliton and the kink (or the anti-kink) governed by b. The latter interaction is attractive because $b > 0$, and therefore it tends to minimize the energy by bringing zeros of ϕ and $\cos(\theta)$ close by. Due to this attraction, the solitons form bound states with either the kink or the anti-kink. As a result, there arise competing forces between the soliton-kink and soliton-anti-kink bound states thanks to the soliton-soliton repulsion and the kink-anti-kink attraction. The final balance depends on the choice of parameters. If the repulsion takes over, (e.g. for $\lambda^2 v^4 \ll a$) the kink and anti-kink will be stabilized at the opposite poles of the circle. This simple example demonstrates how a brane-anti-brane system may become stable in the presence of extra bulk states.

A more realistic example where the stability of the Manyfold has topological origin can be constructed as follows. Let a 3-brane reside in a space with N transverse dimensions, and let ϕ be a real scalar localized on the brane. If its potential has two disconnected degenerate minima, such as in eq. (7.3), selecting a vacuum will spontaneously break the symmetry $\phi \to -\phi$. Therefore there will exist domain walls residing on the 3-brane, which separate the vacua where $\phi = \pm v$. Now suppose that the 3-brane is folded precisely at the location of one such domain wall. The two sides of the fold have the scalar field in a different vacuum. At large distances, the two folds will attract each other due to a massless exchange through the bulk, giving

$$\sim \frac{1}{r^{N-2}}\,, \tag{7.6}$$

which is dangerous since it could collapse the folds onto each other. However, the collapse is incompatible with ϕ having the opposite expectation values on the two folds. This gives rise to a repulsive short-range potential between the folds. Its origin can be deduced as follows: to bring the two folds together, ϕ has to go through zero at each point of one of the folds. But this costs energy $\sim \lambda^2 v^4$, which can be made arbitrarily high by choosing the parameters of the theory. Thus, we expect that at least in a portion of the parameter space, the two folds will be stabilized at some finite distance.[3]

Additional examples can be constructed along similar lines. In string theory, for example, there are non-BPS configurations [31], and they are stable in certain cases. These arguments show that a Manyfold universe may be stable despite the tendencies towards self-annihilation discussed above.

A folded brane is automatically a non-BPS state, breaking all the supersymmetries of the theory. This remains true for D-branes as well as for the field theoretic solitons. The signature of the observed SUSY breaking in the brane spectrum, can be illustrated again by the toy kink-anti-kink brane model above. For simplicity consider a kink domain wall in a $(3+1)$-dimensional Universe. We define the model such that a straight wall is a BPS state. This can be achieved for example in models

[3]Of course in reality the "stuff" that the 3-brane is made of may become important. For example, one can imagine the situation where the brane annihilation can begin before the repulsion sets in. However, concrete examples suggest that the parameters can always be found such that the repulsion can balance the attraction through the bulk.

with a broken discrete R-symmetry, e.g. with the superpotential [13]

$$W = \frac{\lambda}{3}\phi^3 + \mu^2\phi\,. \tag{7.7}$$

This model contains a BPS domain wall, where the supersymmetries emerge because the background admits a central extension of the $N = 1$ SUSY algebra with a central charge which is a difference of the *vevs* of the superpotential at infinities on different sides of the wall [13]:

$$Q_{\mathrm{central}} = 2[W(+\infty) - W(-\infty)]\,. \tag{7.8}$$

This guarantees that 1/2 of the original supersymmetry is unbroken, since the central charge exactly cancels the brane tension in the superalgebra

$$\{Q_\alpha, Q_\beta\} = P_{\alpha\beta} + Q_{\mathrm{central}}\Sigma_{\alpha\beta}\,, \tag{7.9}$$

where $\Sigma_{\alpha\beta}$ is proportional to the area tensor. Thus, there is an exact Fermi-Bose degeneracy in the world-volume theory of the brane.

Now, it is easy to see that a brane-anti-brane system in this theory cannot be a BPS state: the central charge vanishes due to trivial boundary conditions which reflect the topology of the brane-anti-brane configuration.[4] As a result, the Fermi-Bose degeneracy in the brane world-volume theory is lifted. For instance, if branes are stabilized at a finite distance, there is only one massless scalar, corresponding to the center-of-mass motion, but two massless chiral fermions, one on each fold. In this way, an observer living in the Manyfold can never see an unbroken supersymmetry — she will always see an excess of fermionic degrees of freedom. Hence the Manyfold universe could be an interesting source of the observed supersymmetry breaking. It would be interesting to consider in more detail the phenomenological consequences of this "brane-mediated" SUSY breaking scenario.

8. Conclusions

The Manyfold is a natural setting providing us with:

(1) A new dark matter candidate and a novel framework for structure formation;

(2) A mechanism for neutrino masses;

(3) A mechanism for supersymmetry breaking.

[4]If we consider configurations with say an excess anti-brane number, the boundary conditions will be again non trivial. However, supersymmetry can not be restored, since the balance between the central charge and the multi-brane tension is violated. In other words, one can not restore supersymmetry by adding more branes.

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In the limit of infinitely many folds the dark matter becomes dissipationless and replicates the CDM predictions for structure formation. For $n_f \sim 100$ folds dissipation becomes important at the present time and a full study will require a significant effort. Nevertheless therew is a strong possibility that for some range of n_f (or n_{BR}/n_{BL}) and T_R/T_L there will be dark MACHO formation — in the sub-solar mass range — as well as collapses leading to an increase in gravity waves.

The mechanisms and phenomena introduced here, apart from neutrino masses, *do not* depend on having very large new dimensions $\sim$ mm and TeV scale gravity. As long as the bulk is much larger than $n_f \times l_P$ we can use classical higher-dimensional gravity as a good effective theory in which the Manyfold lives. So, even if the string scale is not too far from the old-fashioned M_P, these ideas can be applied for dark matter, supersymmetry breaking and astrophysical implications. The one exception is neutrino masses, where the correct magnitude is reproduced for $\sim$ TeV gravity and sub-millimeter dimensions.

Note added. After the completion of this manuscript, two papers appeared [32] which have some overlap with portions of our section 7.

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